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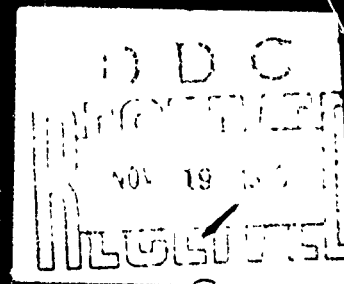
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# A Summary of Methods for Producing Nulls in an Antenna Radiation Pattern

Massachusetts Inst of Tech Lexington Lincoln Lab

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

A SUMMARY OF METHODS FOR PRODUCING NULLS  
IN AN ANTENNA RADIATION PATTERN

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TECHNICAL NOTE 1976-38

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## ABSTRACT

This report presents a brief introductory description of adaptive nulling antenna systems and their essential components, with particular emphasis on satellite communications applications. The principal difference between the inherent performance of array and multiple-beam antennas is discussed. Bandwidth, nulling performance versus degrees of freedom, commonly used circuits, etc. are also discussed. It is pointed out that the algorithm for determining the method for combining the received signals at each of the antenna's  $N$  terminals is a most important and central issue in any system. Several algorithms are discussed and their relationship to the general least mean square (power inversion) algorithm is pointed out. Finally a new method for evaluating the performance of nulling antennas is described.

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## A Summary of Methods for Producing Nulls in an Antenna Radiation Pattern

A recent increase in the use and study of methods to produce nulls, or minima, in the radiation pattern of an antenna motivates a need to summarize the state-of-the-art of known antenna nulling systems. In this summary particular attention is given to these "adaptive" antennas as they may be used in the space segment of a satellite communication system, for the singular purpose of increasing the received signal-to-interference ratio when desired signals operate in the presence of interfering signals. First the fundamental components of an adaptive antenna are discussed. This is followed by a discussion of the classical methods of producing "adaptive" minima and the associated special performance characteristics. Finally the system architecture, its evaluation and expected performance are discussed in general terms.

### Fundamental Characteristics

First let us consider briefly the essential components, the performance characteristics of interest and the canonic forms of adaptive nulling antennas. The basic configuration (i.e., one which contains only the absolutely necessary components) is shown schematically in Fig. 1. The antenna has  $N$  terminals where a signal  $e'_n(\theta, \phi)$  is produced at the  $n$ th terminal by energy arriving from sources located throughout the antenna's field of view (FOV). Here the angles  $\theta$  and  $\phi$  are of a spherical coordinate system which describes the location of the signal sources with respect to the antenna's frame of reference ( $\theta = 0$  corresponds to the center of the FOV) These

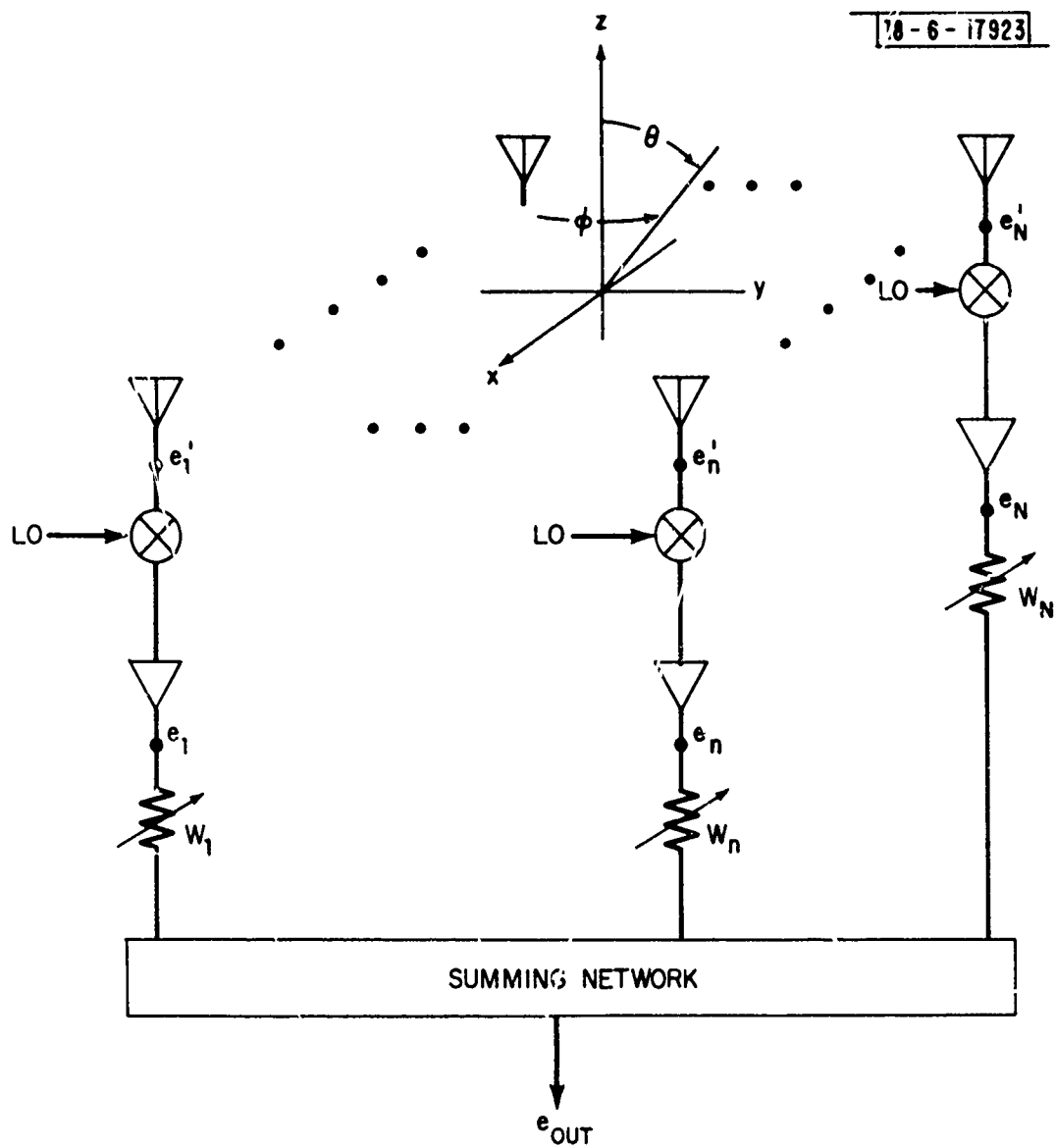


Fig. 1. Basic configuration.

terminals can be associated either with the  $N$  identical elements of an array antenna, or with the beam ports of a multiple-beam antenna (MBA). Each element of the array antenna has essentially the same radiation pattern and identical coverage over the FOV. Consequently, the signals  $e'_n$  produced by an interfering, or user, source located in the FOV have equal amplitude but their relative phases are seldom equal. These same sources produce essentially equal phase and, in general, unequal amplitude signals at the beam ports of an MBA. The significance of this characteristic difference is not obvious now; it will be discussed later. It is virtually the only difference between these basic classes of antennas that may have an influence on their adaptive performance.

The  $N$  voltages  $e'_n$  are usually converted to a different band of frequencies and amplified. The resulting  $N$  signals  $e_n$  are weighted (i.e., multiplied by  $w_n$ ) and summed. Weighting implies changing the relative amplitudes and phases of the signals. Since we are now limiting our interest to receiving antennas, the weights,  $w_n$ , combine to change the effective receiving cross section of the antenna, i.e., its relative response to signals arriving from different directions. (From an alternate point of view, the  $w_n$  determine the radiation pattern if the antenna is considered as transmitting.) These weights are often relatively constant over the antenna's operating frequency band but they can be designed to be "adaptively" varied as a function of frequency by the use of tapped delay lines. The desired value of the weights is derived by an appropriate algorithm. Many algorithms have been defined for determining these weights for known and unknown user and inter-



ference source locations.

In addition to the frequency translation and amplification of  $e'_n$  there is often a need to perform some filtering prior to weighting and summing the resulting  $e_n$ ; however, for our present discussion the "front end" (Fig. 1) is assumed to be made up of  $N$  identical mixers and preamplifiers so as to allow subsequent attenuation without degrading the signal-to-thermal noise ratio ( $S/N$ ) of the  $e_n$  prior to forming their weighted sum.

In addition to the essential hardware indicated in Fig. 1, an algorithm, implemented in analog or digital form, is necessary for determining the weights  $w_n$ . This particular constituent of an adaptive antenna has received and will probably continue to receive a great deal of attention and analytic evaluation. Let us now discuss each component in some detail and then discuss various popular algorithms.

#### Antenna Elements

In an array antenna the elements can be as simple as dipoles or slots arbitrarily located on a planar or non-planar surface. Each element's radiation pattern should cover the FOV so as to enable the enhancement of the antenna's directive gain in the direction of the users. It is common to use elements whose radiation pattern has a main lobe half power beamwidth (HPBW) approximately equal to the angle subtended by the FOV.

The number of elements should be chosen to achieve either the desired directive gain or the required nulling degrees of freedom. When the antenna elements are identical and pointing in the same direction, the array's directive gain can be approximated by

$$G \approx G_0 + 10 \log_{10} N \quad (1)$$

where  $G_0$  is the directive gain of an antenna element in the direction of the observation point; both  $G$  and  $G_0$  are expressed in dB or dBi. For example, ten horns on a satellite at synchronous altitude, each with an earth-coverage radiation pattern and with the weights adjusted to maximize the signals received from a single user, will have a gain between 27 dB and 30 dB depending on whether the user is located at the edge or in the center of the field of view.

It may be necessary to increase  $N$ , above that required to realize the desired directive gain, in order to produce the number of nulls (minima) required to reduce the interfering signals to a tolerable level. In fact, it is common to speak of the antenna's nulling capacity in terms of its degrees of freedom. Generally speaking, an  $N$  element array has  $N-1$  degrees of freedom and can produce  $N-1$  nulls over the FOV. However, this is only true for a single frequency, or narrowband, signal source and for selected locations of the nulls in the FOV. The single frequency constraint can be alleviated by employing weights that vary as a function of frequency (this will be discussed in more detail later). The location constraint is not usually significant if one assumes that the interfering signal sources will be distributed over a substantial fraction of the FOV rather than in a small area within the FOV (say less than 20% of the FOV). Hence,  $N$  is chosen to achieve the desired gain (i.e., by use of Eq. (1)) or at least 1 greater than the expected number of interfering signal sources, whichever is the larger.

When the antenna elements are arranged contiguously so as to form a "continuous" or "filled" array, the resulting radiation pattern (or receiving

cross section) is generally smooth and with nulls (or minima) principally where they are desired. However, for those scenarios where the interfering and user (desired) signal sources are close together, reducing the interfering signals in  $e_{out}$  by placing a minimum in the direction of their sources may cause an intolerable reduction in the antenna's directive gain in the user's direction and the corresponding user's signals in  $e_{out}$ .

Although increasing  $N$  will increase  $S/I$ , this increase in  $N$  may become intolerable in terms of the antenna's weight, power consumption, volume, complexity, etc. Increased  $S/I$  can also be obtained by increasing the size of the antenna aperture while keeping  $N$  constant. This results in an "unfilled" or "thinned" aperture whose radiation pattern will, in general, have more nulls and sharper (narrower) nulls, over the FOV, in comparison to the radiation pattern of a filled aperture with the same number of antenna elements. Unfortunately, these additional nulls may appear in, or near, an area where potential users are located unless particular user locations are specified and  $N$  is greater than the total number of interfering and user sources.

The relationship between antenna aperture and the maximum  $S/I$  possibly poses an interesting problem in physical realizability. Specifically: "What angular separation between a desired and an undesired signal source can be suitably resolved by shaping the antenna's radiation pattern?" is the question commonly asked. Intuitive reasoning quickly leads one to expect that the antenna's aperture size (expressed in wavelengths) must be all important and, not so obviously, the aperture shape is equally important. Using the results of an extensive analysis,<sup>1,2</sup> the directive gain in the

user's direction is reduced  $\approx 3$  dB if a null in the radiation pattern is placed on an interfering signal source and  $\tau$ , the angular separation between the user and the interfering sources, is approximately  $25\lambda/D$  (where  $D$  is the antenna's aperture in the plane containing the satellite and the two signal sources,  $\lambda$  is the operating wavelength and  $\tau$  is degrees). When these sources are closer together (i.e.,  $\tau < 25\lambda/D$ ), the directive gain in the desired signal source direction varies approximately as

$$G' \approx G - 3 - 20 \log_{10} \left( \frac{25\lambda}{D\tau} \right) \quad (2)$$

where  $G$  is the antenna's directive gain in dB when  $\tau > \text{HPBW} \approx 50\lambda/D$  (i.e., from Eq. (1)). Keep in mind, the foregoing values represent the best that can be achieved with a filled aperture; a thinned array will, in general, not quite achieve the indicated performance but it can perform better, in this respect, than a filled array with the same number of elements.

Remembering that we are only considering adaptive antennas for communication satellites, it is quite common for the uplink S/N to be larger than required and that reducing S/I is of prime importance. Consequently, the difference between the antenna's directive gain in the direction of the desired and undesired signal sources is of prime importance so long as the S/N is not intolerably decreased by reducing the directive gain in the desired signal source(s) direction(s).

Any array antenna, and certainly other antennas such as a lens illuminated by an array of feed horns, can be designed to produce a contiguous set of narrow beams which cover the FOV; it is then called a multiple-beam

antenna (MBA). Each beam has essentially the full gain of the antenna's aperture, a HPBW small compared with the angle subtended by the FOV and the beams tend to have a degree of orthogonality in that a signal from one source appears principally at one beam port and not significantly at more than three beam ports. This property of a MBA permits interfering signals to be reduced simply by varying only the amplitudes of only those  $w_n$  for which  $e_n$  contains the undesired signal in significant amount, i.e., by simply turning off those beams which point in the direction of interfering sources. A study<sup>3</sup> of this algorithm for producing minima indicates that greater than 15 dB reduction in the interfering signal can be achieved over more than 85% of the field of view. The width of the minima, or extent of the area over which this reduction in directive gain occurs is, in general, greater than desired, or necessary.

#### Receiver Front End

Although the received signals could be weighted and summed without frequency translation and amplification, it is usually advisable to employ a receiver "front end" as indicated<sup>\*</sup> in Fig. 1. Preamplification is desirable to insure that the attenuation introduced by ensuing operations (frequency translation and weighting) does not degrade S/N. Frequency translation is desirable to reduce coupling between the antenna element terminals (or beam ports) and the output of the summing network. Appropriate filters (not shown) are used to limit the spectrum of signals  $e_n$  to that frequency band  $W_N$  over which effective nulling can be realized.

When antenna pattern shaping (i.e., adaptive nulling) by itself cannot produce the necessary S/I, the communication system must employ additional

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\* The amplifier may also precede the mixer to increase S/N; when preventing small signal suppression is important the amplifier usually follows the mixer.

anti jam (AJ) techniques, i.e., the desired signal is spread over some frequency bandwidth  $W_S$  which is wider than the information bandwidth  $W$ . Pseudo-noise (PN) and frequency hopping (FH) are two common methods of bandspreading for AJ. If the signals are weighted, summed and then despread; the nulling bandwidth  $W_N$  must be equal to or wider than  $W_S$ . With PN spreading it is also important that the antenna's response to the desired signals be relatively constant over  $W_S$ . If  $W_N$  is less than the desired  $W_S$ , it is necessary to despread\* the signals prior to weighting and summing them. Alternatively it may be possible to increase  $W_N$  by using weights that vary as a function of frequency. These weights are discussed in the next section.

If  $W_N$  is substantial, the transfer function of each path should be as nearly identical as possible to assure satisfactory addition of the spectra at the summer's output. It may be possible to compensate for dissimilarities in the mixers, amplifiers or filters by adjusting the weights appropriately but this decreases either the number of interfering signals that can be suppressed or the degree to which they can be suppressed (depth of minima).

### Weights

The weights indicated as  $w_n$  in Fig. 1 operate on the voltages  $e_n$  at each of the antenna's terminals (beam ports) to produce the weighted voltages  $e_n w_n$ . Each weight can change the amplitude and phase of the received signals. In some applications the weighting function is accomplished by introducing attenuation and phase shift as completely separate and independent operations. However, the more common implementation is indicated schematically in Fig. 2. The input signal  $e_n$  is divided into an "in phase" and a "quadrature" component (e.g., by means of a conventional  $90^\circ$  hybrid power divider)

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\*Weighting after despreading a FH spectrum is straightforward; however, weighting after despreading a PN spectrum has doubtful nulling characteristics.

and they are often referred to as the I and Q components. Each of these components is divided into equal amplitude signals by means of a  $180^\circ$  hybrid power divider to result in four equal amplitude signals  $e_{nm}$  ( $|e_{nm}| = \frac{1}{2} |e_n|$ ) which have a relative phase of  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ . These  $e_{nm}$  are then attenuated by  $w_{nm}$  to obtain four signals (i.e.,  $w_{nm} e_{nm}$ ) which are summed to yield the weighted signal output  $e_n w_n$ . There are other versions of this weight circuit but they all employ variable attenuators as opposed to a combination of a variable attenuator and a variable phase shifter. It is particularly important to note that this weight circuit (Fig. 2) varies both the amplitude and phase of the received signals  $e_n$  by the use of fixed power division and phase shifts, and only one variable component, the attenuators. The weights are usually designed to be relatively constant over the operating frequency band. This is primarily because their desired frequency dependence is determined by the location of both the interfering and user source locations both of which are not constant, are probably not known a priori, and will almost always vary throughout the life of the satellite. Consequently, many different variations of the weights, as a function of frequency, will be required during the lifetime of the satellite. A weight that is constant over its operating frequency band is probably a best compromise unless one uses a weight whose frequency variation can be adapted to each user-interference scenario.

However, when the nulling bandwidth exceeds say 10%, and a depth of minima greater than say 30 dB is desired, weights with adaptive frequency dependence must be employed. Such a device can be realized by the weight circuit schem-

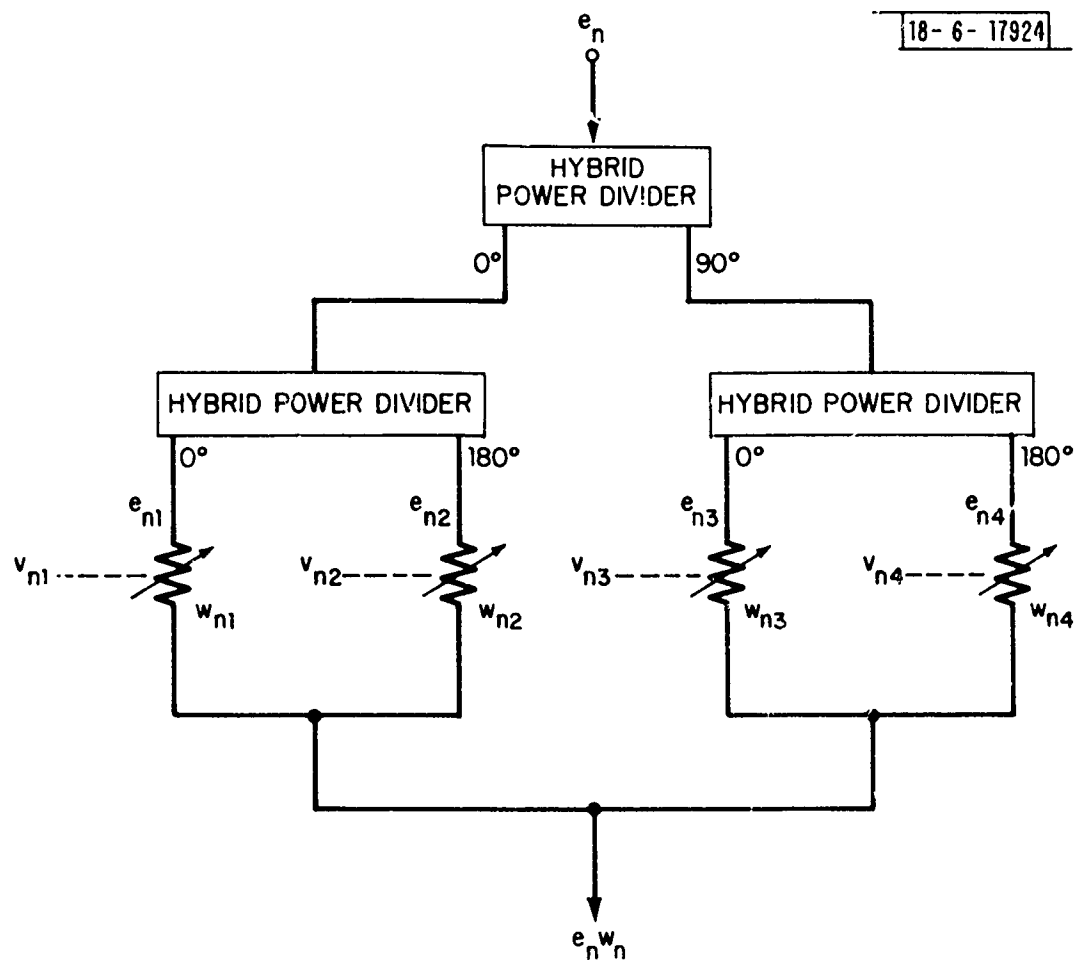


Fig. 2. Typical weight circuit.



atically shown in Fig. 3. Several frequency independent weights of the type shown in Fig. 2, are connected to the taps on a delay line which is carrying  $e_n$ . The signal out of each tap (i.e.,  $e_{np}$ ) is weighted by  $w_{np}$  to give  $P$  weighted versions of  $e_n$  (i.e.,  $e_{np} w_{np}$ ) which are summed to produce  $e_n w_n$ . Although the  $w_{np}$  do not vary with frequency, the  $e_{np}$  have a relative phase which varies with frequency due to the differential delay the signal  $e_n$  undergoes in propagating along the delay line. Considering a single interfering signal source located at the edge of the FOV, the electrical length ( $L$ ) of the delay line need not be any longer than the maximum differential electrical path length between the signal source and the edges of the antenna aperture. For a synchronous satellite

$$L = 0.15 D\beta \quad (3)$$

where  $\beta$  is the ratio of the wavelength on the delay line to that in free space (usually  $\approx 0.67$ );  $D$  is the diameter of the antenna's aperture. The taps should be spaced so as to suitably approximate the desired delay regardless of the signal location in the FOV. In other words the tap spacing should be chosen so as to result in a variable delay line with resolution suitable for the desired performance.

However, if more than one interfering signal source are in the FOV, the length of the delay line and the tap spacing,  $S$ , must be chosen on the basis of a more sophisticated analysis. Generally speaking,  $S$  must satisfy the relation

$$S \leq 300/W_N \quad (4)$$

where  $S$  is in meters and  $W_N$  is in MHz. The number of taps can be approximated

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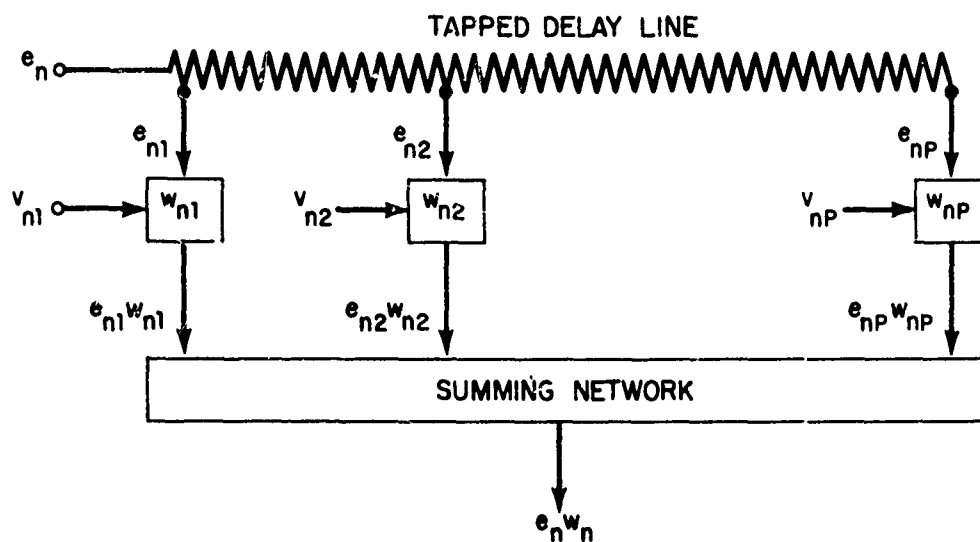


Fig. 3. Frequency adaptive weights.

by

$$P \geq W_N/W_0 \quad (4a)$$

where  $W_0$  is the nulling bandwidth over which frequency independent weights produce satisfactory results.

For purposes of increasing  $W_N$ , these frequency adapting weights can be considered to increase the antenna's degrees of freedom to a number equal to the product of the number of array elements (or beam ports of an MBA) and the number of taps on the delay line of a frequency adaptive weight (i.e.,  $NP$ ). The required degrees of freedom (DOF) is approximately equal to  $1/2$  the product of the number of interfering sources ( $N_I$ ) and the percent frequency bandwidth over which instantaneous nulling is desired (i.e.,  $DOF = 50 N_I (W_N/W_0)$ ). However, one cannot substitute delay line taps  $P$  for antenna elements  $N$ ; that is  $N$  should be greater than the number of interfering sources. It also follows that when  $N$  exceeds the number of interfering sources, the additional DOF might increase  $W_n$  or increase the user's signal strength.

#### Weight Determining Algorithms

In order to achieve the desired antenna performance an algorithm for choosing the  $w_n$  must be formulated. These systems range from the very simple and straightforward to the complicated and sophisticated. They all must derive the desired  $w_n$  and ascertain that it is properly installed. It is helpful to separate these algorithms into two general classes--those with and those without feedback. Specifically the latter, or feed forward systems, determine the desired  $w_n$ , from a given set of input data, and install them

with some small but finite error. They are not further corrected by examining the output signals as is done in an algorithm which uses feedback. The weights in antenna system such as the currently planned DSCS-III and JARED antennas are derived from input data involving locations of signal sources and the antenna's response characteristics. They are implemented by command voltages  $V_n$  supplied to the weights. It is not planned to sense the output for the purpose of fine tuning the weights. Hence it is common to refer to the weights as "commanded".

Our primary interest here is in adaptive algorithms, i.e., algorithms in which the weights are adjusted by means of feedback control. The algorithm for deriving weights is undoubtedly the central issue in determining the value and performance of any adaptive antenna system. This is especially true if the algorithm cannot be significantly modified when the satellite is operational. More will be said about this later; let us now consider the all important weight determining algorithms.

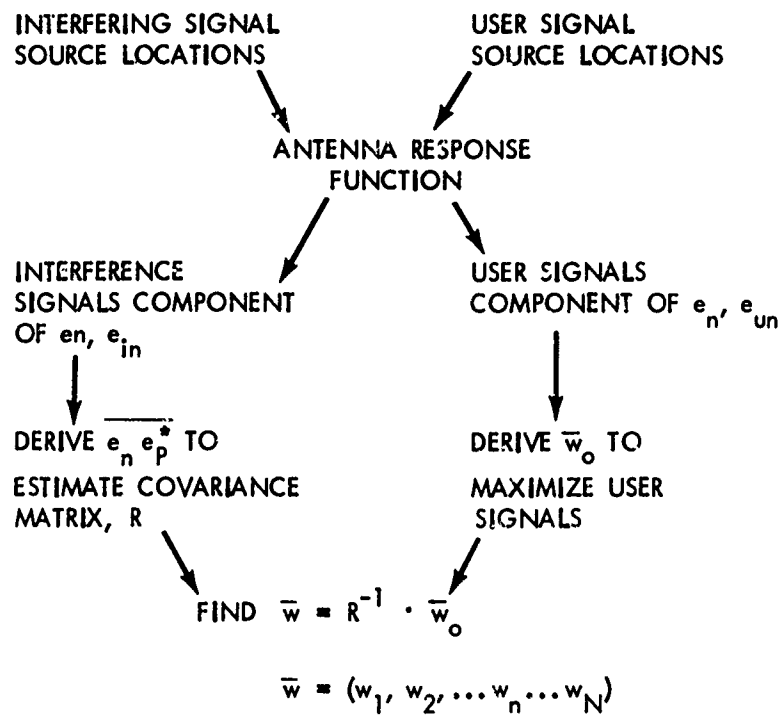
Let us first consider the simplest algorithm and then proceed along a general approach to the more sophisticated methods for deriving  $\bar{w}$  which is a "vector" notation for the set of weights  $w_1, w_2, \dots, w_N$  (i.e.,  $\bar{w} = (w_1, w_2, \dots, w_N)$ ). Let us further consider principally those cases where the interfering signal power is much larger than the user's signal power, when both are measured in the nulling band  $W_N$ . This assumption is valid since adaptive nulling without bandspreading does not usually provide sufficient S/I enhancement.

If the weights of a MBA are chosen such that  $w_n = 1$  when  $|e_n| < A$  and  $w_n = 0$  when the  $|e_n| \geq A$ , the antenna's "radiation pattern" will cover the FOV except for areas surrounding the interfering signals of level  $> A$ . Analysis<sup>3</sup> has shown that this algorithm will choose weights of a particular MBA that will reduce the interfering signals by more than -15 dB regardless of their location in the FOV. If one wishes to reduce the interfering signal more than 15 dB, the choice of threshold power level  $A$  may become intolerably critical and the area over which the antenna's gain is reduced may be large enough to degrade or disable communication with users located near the interfering source(s). Consequently, this simple algorithm may not yield the satisfactory results which the antenna may be capable of producing.

Next let us separate the algorithm into two basic and distinct parts. First recall that the voltages,  $e_n$ , at each antenna terminal are produced by thermal noise and interfering sources and user sources in the FOV. That is  $e_n = e_{\text{noise}} + e_{\text{in}} + e_{\text{un}}$ , respectively. The goal of any algorithm is to choose the weights so as to minimize the total power received from the users so as to realize an output voltage  $e_{\text{out}}$  dominated by the user's signal. Since these three signal sources are uncorrelated with one another, the covariance matrix  $R$ , formed by determining the time average value of  $e_n e_q^*$  (i.e.,  $\overline{e_n e_q^*}$ , where  $*$  means complex conjugate value) equals the sum of the covariance matrices of the thermal noise, interfering signal sources and user sources; that is  $R = R_{\text{noise}} + R_I + R_u$ . In most communication satellite systems, the interfering signals dominate  $R$ , (i.e.,  $R_I \gg (R_u + R_{\text{noise}})$ ) and a good estimate of  $R$  can be obtained from the voltages  $e_n$ . In the absence of

interfering signals (i.e.,  $R_I = 0$ ) the weights  $\bar{w}_0$  are chosen to maximize the signals of all the users. Choosing the desired weights  $\bar{w}$  equal to the vector product of the inverse of the covariance matrix  $R$  and  $\bar{w}_0$  will maximize the signal-to-total noise (i.e.,  $e_u / (e_i + e_{\text{noise}})$ ) ratio in the output voltage  $e_{\text{out}}$ . This maximization will occur for all user signals in accordance with the initial choice of  $\bar{w}_0$ .

Let us consider a common scenario to illustrate this basic algorithm. Assume that all signal source locations and the relative intensity of each users' effective radiated power (ERP) are known. Let us further assume that the antenna's response to a signal source located anywhere in the FOV can be calculated. Following the outline given in Fig. 4, the user and interference signal components of  $e_n$  can be calculated from the foregoing information. The covariance matrix  $R_I$  and its inverse can be calculated from  $e_{in}$  and a least mean square fit, or an iterative algorithm can be used to calculate that  $\bar{w}_0$  which will minimize the difference between the calculated and the desired antenna directive gain in the direction of the users. Choosing the weights  $\bar{w}$  equal to the product  $R_I^{-1} \cdot \bar{w}_0$  will maximize  $S/I$  in the case of a single user. In the case of multiple users, a minimum will be placed in each interfering signal source direction and simultaneously the quantity  $(\bar{w} - \bar{w}_0)^2$  will be minimized. The latter condition is equivalent to minimizing the difference between the actual and the desired antenna directive gain in the direction of the users. If there is only one narrowband interfering source in the FOV, the depth of the minimum will, in general, be  $\approx 2X$  dB when the interfering signal power is  $X$  dB above thermal, or effective, noise at the



NOTE  $e_{in} + e_{un} = e_n$

Fig. 4. Basic algorithm.

input to the front end. If the constraints (i.e.,  $\bar{w}_0$ ), the number of interfering sources and the nulling bandwidth require degrees of freedom in excess of those available,  $\bar{w} = R^{-1} \cdot \bar{w}_0$  will still maximize S/I; however, the magnitude of S/I will be smaller.

All adaptive nulling algorithms, in their steady state, attempt to choose  $\bar{w} = R^{-1} \cdot \bar{w}_0$ . Consequently, specific performance differences among various algorithms are manifested in their transient behavior, their choice of  $\bar{w}_0$ , the hardware and/or software implementation of them and the degree to which they are sensitive to errors.

#### Minimization of Received Power

Let us next assume that the location and strength of the interference signals are unknown but they are much stronger than the users' signals (i.e.,  $R_I \gg (R_u + R_{\text{noise}})$ ). This would be a common scenario in the DSCS environment. As before, the  $\bar{w}_0$  are selected to provide the desired antenna directive gain in the known direction of each user. Since the location of the interfering sources is postulated not to be known, their contribution to  $e_n$  cannot be calculated. Direct measurement of the  $e_n$ , or  $\overline{e_n e_p^*}$ , would enable estimating  $R$  and  $R^{-1}$ ; this estimate becomes more accurate as the interfering signals become much stronger than the user signals. Choosing the nulling bandwidth appropriately can guarantee that troublesome interference will produce a large I/S and the algorithm will be very effective in choosing  $\bar{w}$ . Conversely, an inappropriate choice of nulling bandwidth, effective noise level, etc. will result in an undesirable reduction in the user's signals if it (they) and the interference signal(s) have approximately the



same amplitude (i.e.,  $|e_{in}| \approx |e_{sn}|$ ). Consequently, the algorithm can differentiate user source from interfering sources only on a relative amplitude basis.

This algorithm is often referred to as the "power inversion" or Applebaum-Howells algorithm.<sup>5</sup> It is one of the best known analogue algorithms. A schematic representation of this circuit is shown in Fig. 5. The antenna element, or beam port, output signals  $e'_n$  are indicated for an N element (beam) array (multiple-beam antenna). A mixer followed by a preamplifier (and perhaps appropriate filtering) establish  $e_n$  over the nulling band  $W_N$ . For the purpose of the present discussion let us assume that  $e_n$  is a frequency translated frequency band limited representation of  $e'_n$  and consider the "loop" that is connected to antenna terminal #1. As indicated in Fig. 1, the signals  $e_n$  are weighted by  $w_n$  and summed to give an output signal  $e_o$ , that is

$$e_o = \sum_{n=1}^N w_n e_n. \quad (5)$$

Thus far everything is exactly as described in the previous sections. However in this circuit the complex weights  $w_n$  are proportional to the complex control voltages  $V_n$ . As in any adaptive algorithm the derivation of  $V_n$  is our present interest. Note that the signal  $e_1$  is mixed with  $e_o$ . The output of the mixer is low pass filtered and amplified giving a complex voltage proportional to correlation of  $e_1$  with  $e_o$ . The correlator's output is subtracted from a beam steering voltage  $B_1^*$  to give  $V_1$ . For the moment let us assume  $B_1^* = 0$ . If  $e_o$  is correlated with  $e_1$ , the low pass filter integrates the output of the correlator to produce  $V_1 \neq 0$  which changes the weight  $w_1$

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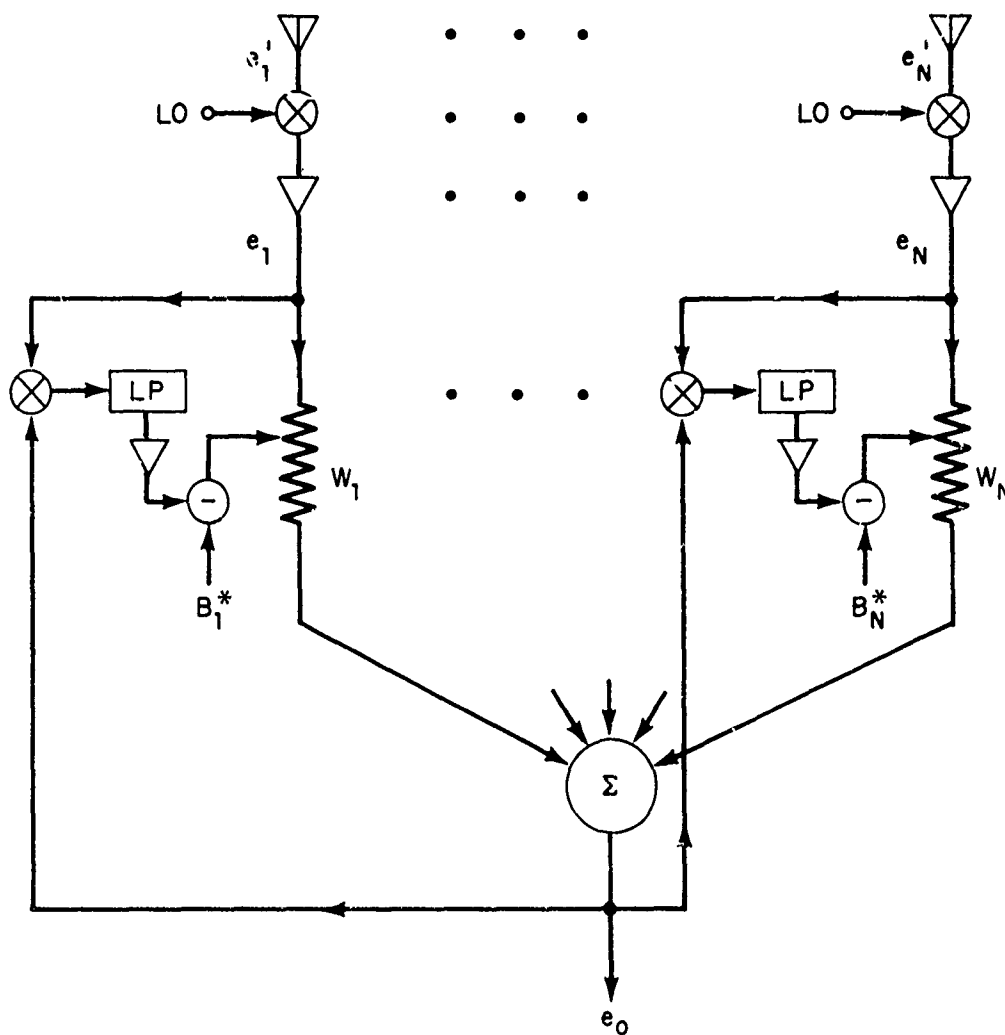


Fig. 5. Applebaum-Howell circuit.

so as to reduce the correlation between  $e_0$  and  $e_1$  which in turn reduces the output of the correlator. Similar response in the other loops reduces  $e_0$  to a minimum. Noise in the circuit and/or in  $e_1$  prevents  $e_0$  from vanishing. Hence we see that any signal (in  $e_1$ ) above the effective noise level will be sensed by the loop and reduced below the effective noise level. Furthermore, in the absence of signals (interfering or user) above the front end noise level, the weights will be determined by the noise. However, if  $B_n^*$  are not zero (i.e.,  $\bar{B}^* \neq 0$ ), they will determine the weights (i.e.,  $\bar{w} = \bar{B}^*$ ) and the antenna pattern will assume its quiescent, or desired, shape if we set  $\bar{B}^* = \bar{w}_0$ ; we determine  $\bar{w}_0$  a priori from known or expected user locations, etc.

It can be shown<sup>4</sup> that, when  $R_I \gg R_{\text{noise}} > R_u$ ,  $R \approx R_I$  and the steady state weights assume the value  $\bar{w}_n = R_I^{-1} \cdot \bar{B}^*$ . Hence,

$$\bar{w}_{\text{opt}} = R_I^{-1} \cdot \bar{w}_0 \approx R^{-1} \cdot \bar{B}^* \quad (6)$$

where  $\bar{w}_{\text{opt}}$  is the optimum set of weights. The transient performance and attaining stable steady state weights that equal  $\bar{w}_{\text{opt}}$  are of paramount concern and will be discussed later. The presence of a feedback loop (i.e., correlation of  $e_n$  with  $e_0$ ) places this algorithm in a class different from those discussed previously. The inherent tendency of a feedback circuit to correct for its imperfections makes this advantageous in a "hands off" environment such as on board a communication satellite.

In summary the Applebaum-Howells circuit establishes  $\bar{w}_{\text{opt}}$  when  $\bar{B}^* = \bar{w}_0$  and the interfering signals are large compared to the user signal (i.e.,  $R \approx R_I$ ). Thus the user locations and the antenna response function

must be known and used to determine  $\vec{B}^*$ . It is also necessary for the level of the users' signals to be approximately equal to or less than the front end noise level (i.e.,  $R_S \leq R_N$ ). This circuit senses the location of all interfering signals and reduces them below the front end noise level (both are measured in the nulling band). If the interfering source is close to a user, the latter's signal will also be reduced but the ratio  $e_I/e_u$  will not be increased; it will probably be decreased.

#### Maximization of Signal-to-Noise Ratio

When there is a single user present in an environment of one or many interfering sources, the definition of S/N is unambiguous. This is not true when multiple users are present; however, the Applebaum-Howells circuit allows for optimum adaption when several users and several interfering sources are present. When there is only a single user and his location is known, the Applebaum-Howells circuit will maximize S/N if the quiescent weights  $\vec{B}^*$  are chosen to form a single maximum directivity beam pointing in the direction of this user. The circuit shown in Fig. 6 will form and/or steer a high directivity beam in an unknown user's direction while simultaneously placing a null on all interfering signal sources. This is accomplished without a priori knowledge of the location of either the user or the interfering sources. However it is necessary to derive a reference signal that is a suitable replica of the user's signal. This can lead to the dilemma that if the user's signal must be known a priori what is the need to send it to the satellite receiver. However, the user's signal might have a deterministic component (i.e., a pseudonoise or frequency hopped "carrier" known a priori)

and a random component (i.e., the modulation on the carrier). The former would be used to permit the adaption circuit to maximize S/N.

The circuit shown in Fig. 6 is identical to the Applebaum-Howells circuit (Fig. 5) discussed in the previous section except for the output portion and the  $\bar{B}^*$  weights. A reference signal  $e_{REF}$  is subtracted from  $e_o$  to generate an error signal with which the  $e_n$  are correlated and the  $w_n$  determined. In this circuit the reference signal was obtained by despreading  $e_o$  and bandpass filtering it in order to increase the S/I of  $e_{REF}$  compared to the S/I of  $e_o$ . Amplifying the demodulated  $e_o$  (i.e., to obtain  $e_{mess}$ ) and modulating it with the known pseudonoise sequence results in a reference signal,  $e_{REF}$ . To understand the operation of this circuit let us assume that, in  $e_o$ , the interfering signal strength is much larger than that of the user's signal strength. Consequently, the error signal is dominated by the interference signal and the correlators drive the weights to reduce  $e_o$  by placing nulls in the direction of the interfering sources. Reduction of the interfering source will not, in general, reduce the users' signal; consequently the reference signal makes up an increased portion of the error signal. Because the reference signal is principally an amplified replica of the user's signal, the correlators drive the weights to increase the users' signal. This adjustment of the weights is identical to steering a maximum directivity beam toward the user while simultaneously placing nulls in the direction of the interfering sources.

This circuit (Fig. 6) is commonly referred to as the "Widrow" circuit after its inventor.<sup>6</sup> It is often recommended for use in time division

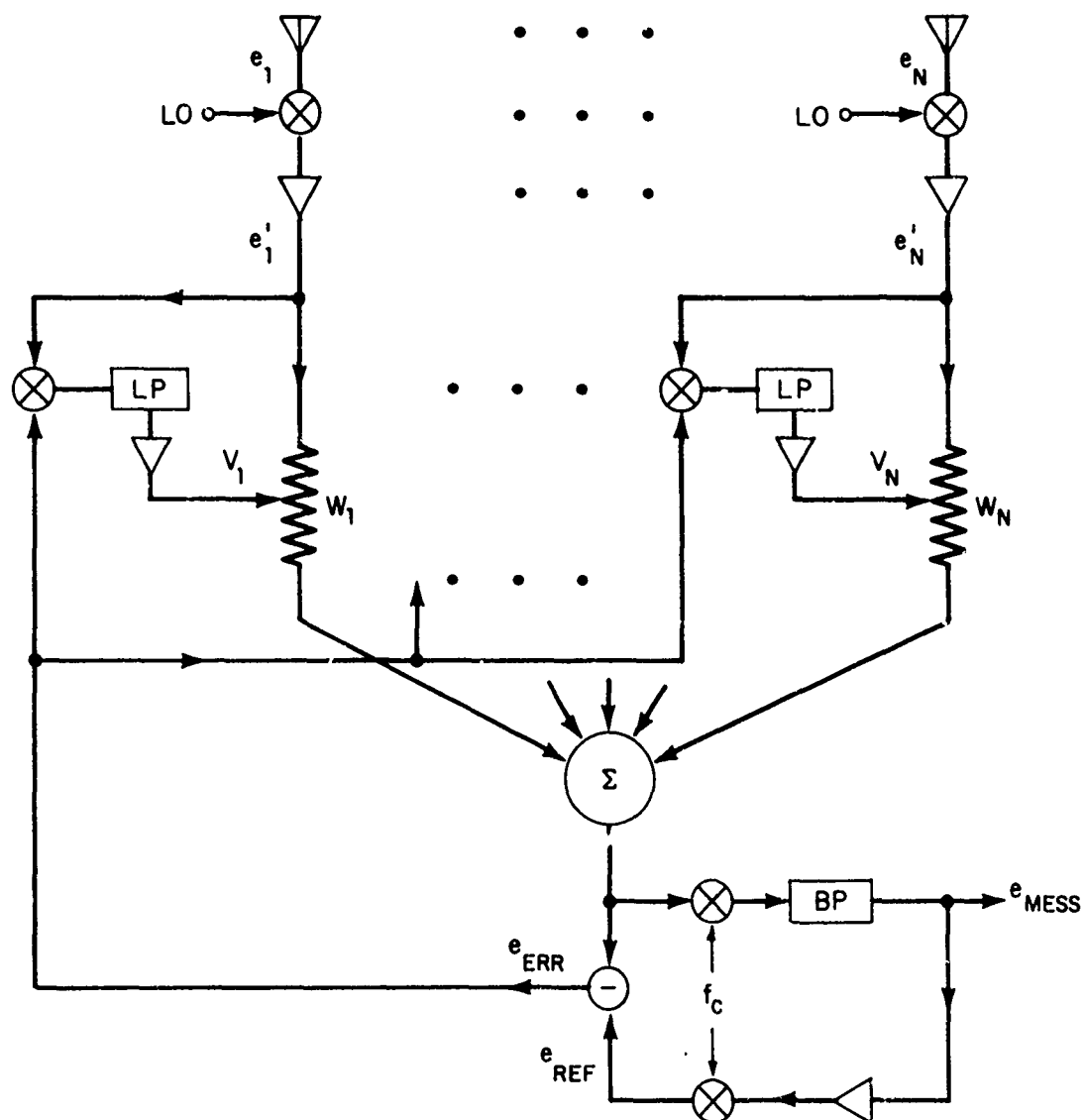


Fig. 6. Widrow algorithm.

multiple access (TDMA) communication systems employing the pseudonoise method of bandspreading. It is limited to single user multiple-interference sources scenarios because with more than one user present, the beam will be acquired by only that user with the largest ERP.

It is interesting to note that the Applebaum-Howells and Widrow circuits produce the same steady weights if the  $\bar{B}^*$  are chosen to steer the beam in the user's direction. That is, the Widrow circuit contains a closed loop determination of the  $\bar{B}^*$ ; whereas the Applebaum-Howells circuit requires that they be determined open loop. Consequently, the former is better for a single user but the latter is better suited toward generating a prescribed quiescent radiation pattern such as an earth coverage or multiple area coverage pattern to serve multiple users simultaneously.

#### Transient Characteristics

Any of the algorithms discussed requires a finite time to establish its weights. The analogue circuits have finite loop time constraints and the computational systems require a finite computational time. Let us discuss 1) why transient response is of interest, and 2) the transient phenomena associated with the analogue circuits. Then we will show how a computational method permits the use of a non-real time processing technique which essentially eliminates transient performance of the adaptive circuits.

In a TDMA communication system, or in one which uses frequency hopping as a bandspread modulation, the adaptive circuits must reach their steady state in a time period short compared with the access or the hopping rates. For example, in a TDMA system with a dwell time of 20 msec. per user, the adaptive circuits should reach their steady state in less than 2 msec.

A similar maximum adaptive time would be tolerable if the hopping rate were 50 hops/sec.

It can be shown that for a single interfering source the time constant of the  $n$ th cancellation loop (Fig. 5 or 6) is inversely proportional to the product of the loop gain and the received power  $|e_n|^2$ . The loop gain must be set as high as possible to reduce the adaption time but it must not be so high as to cause loop instability. Therefore setting the loop gain in accordance with the strongest expected interference signal power prevents loop instability but also determines the weakest interfering signal that the circuit will adapt to in a given time. Unfortunately, it is not only the weaker source that is "nulled" slowly. The strongest source is at first reduced rapidly to a level somewhat higher than its steady state level. Then at a much slower rate its signal strength is reduced until all interfering signals are reduced to their steady state level at the same time. Consequently, it is important to set the loop gain so that the weakest expected interfering signal strength will result in a tolerable loop time constant. If the spread in strength of all troublesome interference sources is sufficiently large it may not be possible to choose a suitable loop gain. This situation is often referred to as an excessive spread in the eigenvalues of the covariance matrix  $R$ . Fortunately there are several ways to modify these circuits to reduce the spread in the eigenvalues.

In a previous section the salient difference between a MBA and array antenna related to the fundamental character of the terminal or beam port signals  $e'_n$ . It was pointed out that with a MBA the  $e'_n$  are essentially in phase or about  $180^\circ$  out of phase with one another; whereas, their relative ampli-



tudes varied widely. For example, a signal source located in the  $i$ th beam of a MBA will result in  $|e_i|$  much larger than all other  $|e_n|$  and the relative phase of all  $e_n$  will be  $\approx 0$ , or  $180^\circ$ . If a second source is located in the  $j$ th beam  $|e_i/e_j|$  will be approximately equal to the ratio of the strength of the two sources and would probably be closely related to the ratio of the largest to the smallest eigenvalues of the covariance matrix  $R$ . Preweighting  $e_i$  by  $w_{pi}$  will result in a new ratio of signal levels  $|w_{pi} e_i/e_j|$  which will in turn compress the spread in eigenvalues by approximately  $w_{pi}$  and of course reduce the spread in the adaption time constant. A preliminary study indicates that preweighting is particularly effective when the interfering sources are separated by more than a beamwidth.

Probably the most effective method of compressing the spread in loop time constants or eigenvalue spread is by using a preprocessor between the outputs of the front ends and the inputs to the cancellation circuits. The function of the preprocessor is to "equalize" the eigenvalues by creating a "quasi-MBA" whose "beams" are determined by the eigenvectors of  $R$ , and employing an automatic gain control in the output of each "beam" port.

In order to better understand this, observe, that any eigenvalue  $\lambda_i$  of  $R$  corresponds to the relative power,  $P_i$ , that would be delivered to the output of the summing network if the weights (i.e.,  $w_1, w_2, \dots, w_N$ ) were set equal to the components of the  $i$ th eigenvector of  $R$ . If we identify  $\lambda_1 > \lambda_2 > \dots > \lambda_N$ , choosing  $\bar{w}_1$  equal to the components of the eigenvector corresponding to  $\lambda_1$  will result in maximizing the signal  $e_o$  out of the summing network. It is also true that the radiation pattern would point a beam in the direction of

the largest source in the FOV if there is one source substantially larger than the others. If there are two, or more, largest sources of equal intensity,  $\bar{w}_1$  may form a multiple-beam pattern pointing toward these largest sources. In short  $\bar{w}_1$  will form that pattern which will collect the most energy from the sources in the FOV (the "eigenbeam" associated with  $\lambda_1$ ). If the weights are chosen equal to  $\bar{w}_2$ , the components of the eigenvector corresponding to the  $\lambda_2$ , they will form a radiation pattern with a minimum in the direction of the strongest source and a maximum, generally, toward the next largest source (the eigenbeam associated with  $\lambda_2$ ). The depth of the minimum and the discrimination between strongest, not so strong, etc. signal sources depends on the spread in the eigenvalues. However, it is most important to note that power collected when  $\bar{w}_1$  is installed is not available to any other  $\bar{w}_n$  that is equal to the components of the eigenvector corresponding to  $\lambda_n$  ( $n \neq 1$ ) of  $R$ . Similar statements can be made for  $\lambda_2 \cdots \lambda_N$ ,  $\bar{w}_2$ , etc. Therefore the preprocessor sequentially separates the received power,  $P_{11}$  associated with each  $\lambda_n$  and delivers it to one of  $N$  terminals. If an automatic gain control  $g_n$  maintains the spread in these levels within the desired limits, the output signals  $e_{on} = \sqrt{P_n} g_n$  will have a covariance matrix whose spread in eigenvalues is tolerable. A practical implementation of a preprocessor that performs this function exists.<sup>7</sup> A MBA performs a function somewhat like the preprocessor just described in that its beams tend to be orthogonal and they span the FOV.

### Digital Implementation

Although the circuits shown in Figs. 5 and 6 indicate the use of analogue devices, it is not uncommon to use digital devices between the output of the correlator and the control terminal of the weights. Within the scope of this report these changes do not modify the preceding discussions. However, consider a processor which converts the antenna terminal voltage  $e_n$  to a digital representation and completes the adaption process etc. entirely within a computing facility. For example, the processor indicated in Fig. 7 first reduces the instantaneous bandwidth of the receiver by dehopping or inserting the pseudonoise sequence. The intermediate bandwidth is chosen to retain the necessary power differences between interfering and user signals. The signals are amplified, to establish the system thermal noise level, and divided into I and Q channels. The signals are then reduced to baseband and divided into a narrowband "signal" band and a wider "nulling" band. All signals are digitized, the wide band signals are used to compute the covariance matrix and/or the optimum weights. The narrow band signals are stored momentarily, while the weights are being computed. They are then weighted, summed and filtered to yield a digital representation of the user(s)' signals. Notice that this processor eliminates the transient performance of the weights and has the linearity characteristics of a digital computer. It has the characteristics of: 1) deriving the weights from signals whose bandwidth is characteristically less than  $\approx 0.5$  MHz due to the limited speed of current A/D devices, 2) requiring the dwell time, in either a TDMA or frequency hopping mode, to be larger than the time to compute the weights, 3) requiring a

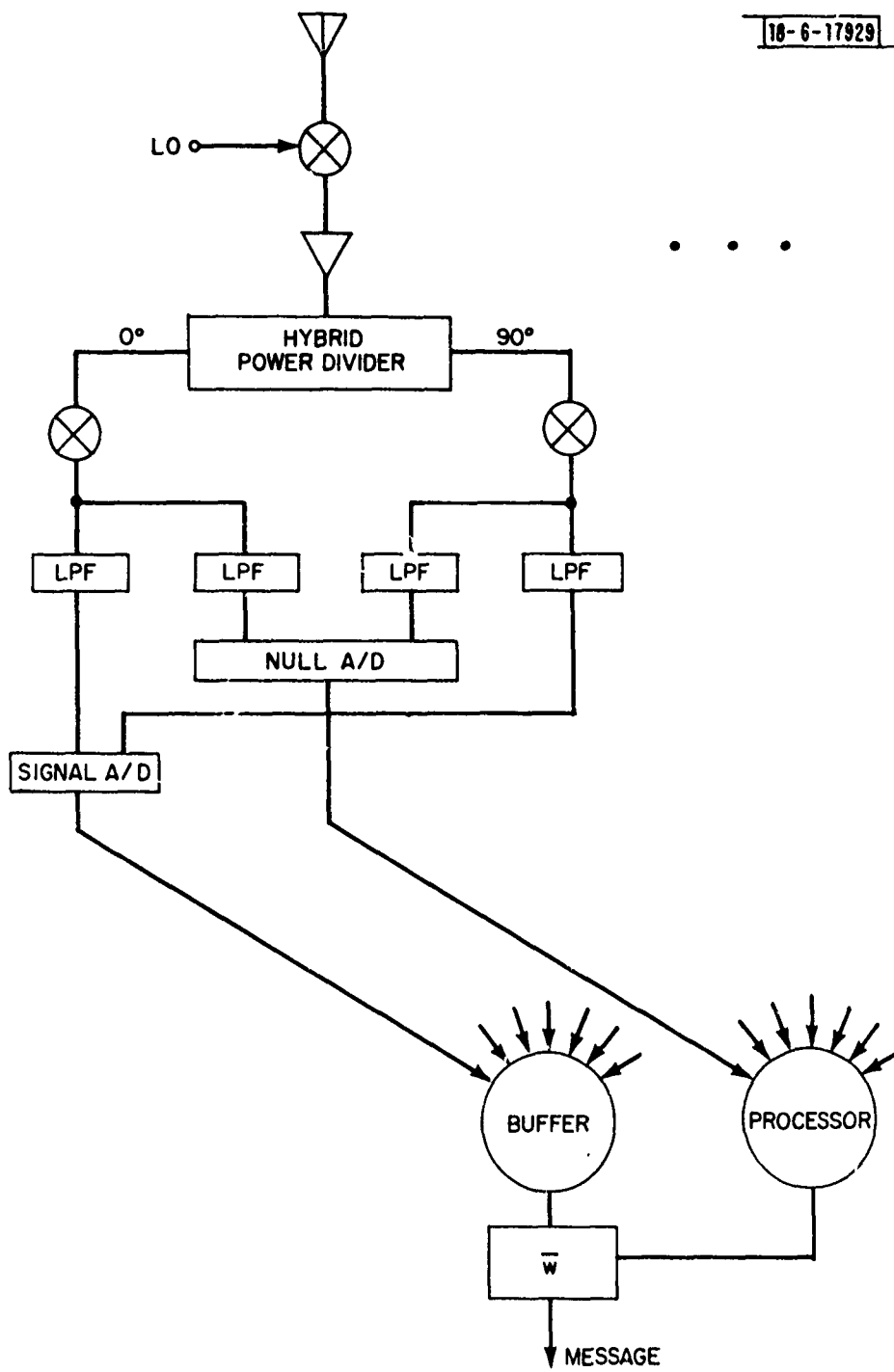


Fig. 7. Completely digital algorithm.

non-trivial storage medium (buffer) and computational facility. Since the transient phenomena has been eliminated, the eigenvalue spread may not be an important factor. If it is important, a "digital" preprocessor can be implemented in the computational algorithm.

#### Nulling System Architecture

A maximum degree of "electronic" and "physical" survivability will accrue if the adaptive circuitry etc. is on board the satellite. This increases the complexity, weight, power consumption, etc. of the satellite but it does achieve secure autonomous operation.

Alternately, the array element, or beam port, signals could be transmitted to earth via a high data rate secure channel and the "adaptive receiver" could be completely implemented on earth. The demodulated user's signals could then be transmitted to the satellite via a secure link for ultimate transmission to the intended receiver. This system architecture has the advantage that its "receiver" and adaptive algorithm can be modified or completely changed during the lifetime of the satellite. It has the disadvantage of requiring secure and survivable satellite-to-earth and earth-to-satellite links and having a vulnerable failure point in the form of the earth station. The fidelity of the transmitted array element signals could also be of concern if appropriate signal bandwidth or satellite ERP is not available. An  $N$  element system requires a bandwidth at least  $N$  times the desired instantaneous nulling band. Increasing the array's size, or the number of beams in the MBA, could lead to the requirement for an intolerably large bandwidth and/or ERP on the downlink.

Systems that are hybrid with regard to the previous two, do not allow for

closed loop operation (i.e., because of 1/4-second round trip delay time between the earth and a synchronous satellite). Further discussion of these systems is beyond the scope of this report.

#### Evaluation of Nulling Systems

The uncertain location of the interfering sources and somewhat indefinite location of users with respect to the satellite's frame of reference render the evaluation of any adaptive antenna's performance ambiguous if only a few "representative" user-interference source scenarios are considered. For example, the thinned array characteristically has more nulls than are required. If these extraneous nulls are not constrained by the antenna system's algorithm, a carefully chosen scenario can lead to performance that misrepresents the antenna system's capability. It may also be possible to design the antenna system and its algorithm so that it will perform very well when operation is confined to a given scenario or set of scenarios. Furthermore evaluation of the antenna performance to a given user-interference source location is somewhat subjective, even when a contour plot presentation of the S/I improvement is the basis for judgement. For these reasons, we now consider a statistically based method for evaluating the performance of any commanded or adaptive nulling antenna system.

The system designer when computing the channel, or link, margin needs to know the signal strength produced, at the adapted antenna's output, by the user and interfering sources. Since thermal noise is essentially unchanged by varying the weights to improve S/I, knowledge of the antenna's directive gain in the direction of all signal sources (in the FOV) enables

calculation of output signals produced by both the user and the interfering sources. Most scenarios involve point interfering sources and areas where several users may be located; hence a contour plot of the adapted antenna's directive gain, over those areas where the users are located and those smaller areas or points where the interfering sources are located, provides the system designer with the needed information to determine link, or channel, margin, etc. Specifically, this contour plot enables the performance of the antenna, operating under a given interference stress, to be evaluated for several user scenarios--or at least for the original scenario used to derive the weights.

It is usually agreed that interference sources of significance can be located essentially anywhere in the FOV. An equally significant consensus indicates the users will be confined to specific areas distributed randomly over the FOV. However, to this date a suitable set of user areas has escaped definition. Pending the generation of "typical", or "representative", user scenarios (i.e., both location and ERP) it seems reasonable to consider users to be randomly located anywhere within the FOV. Hence let us assume that the quiescent radiation pattern of the antenna system covers the FOV (i.e., it is an earth-coverage pattern).

With the foregoing in mind, we define a Figure of Merit, indicating the adaptive antenna system's performance, on a statistical basis which gives the probability that the adapted antenna's directive gain, over the FOV, exceeds the directive gain in the direction of the interfering sources by a given value. The probability of achieving a given S/I can then be calculated

from the known user and interference source strengths effective over the nulling frequency band. This statistical distribution of directive gain can be derived from the radiation patterns of the antenna system adapted in response to many user-interference source scenarios.

For example, let us assume that many users and say five interfering sources exist within the FOV. Random locations for the interference sources are specified and an earth-coverage quiescent pattern is used to determine  $\bar{w}_0$ . Either experimentally or analytically the antenna system is allowed to adapt and the directive gain is determined at many uniformly distributed points over the FOV. This process is repeated for, say, 25 scenarios. In its simplest form the Figure of Merit would be obtained by calculating the cumulative distribution of the directive gain at these points in the FOV and plotting it as indicated in Fig. 8. In addition, the cumulative distribution of the directive gain in the interfering source direction is plotted as shown in Fig. 8. (Strictly speaking the area on the surface represented by a point in the FOV would have to be taken into account but this is straightforward and not necessary to the present discussion.) Information can be derived from this graph in the following manner:

1. There is a 0.75 probability that the directive gain, to a user will exceed 10 dB.
2. There is a  $(1-0.75) = 0.25$  probability that the directive gain to an interfering source will be less than -38 dB.
3. The probability that the S/I ratio exceeds  $10 - (-34) = 44$  dB is at least 0.75.

This information is not only useful in designing the satellite communication system; it is particularly useful in evaluating the performance of one



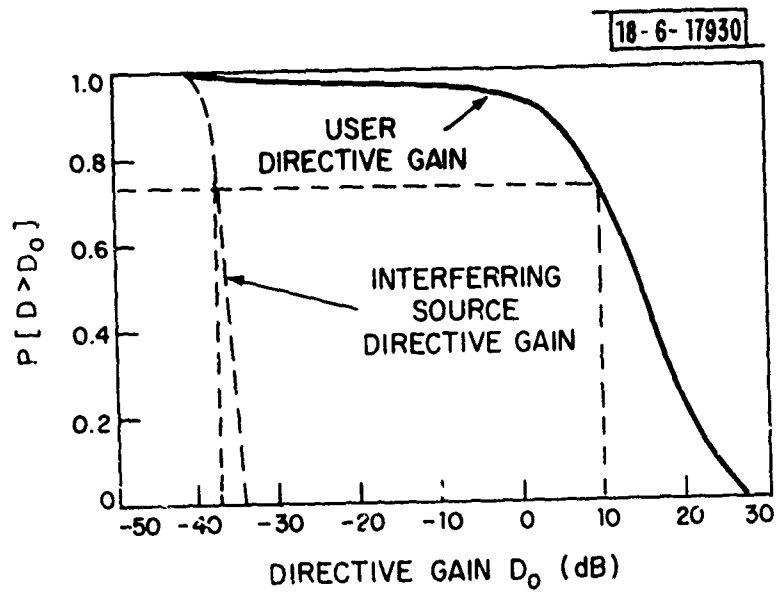


Fig. 8. Directive gain  $D_0$  (dB).

adaptive antenna system versus the performance of a competitor.

In the case of a more specific user scenario only the directive gain in the areas, or at the points, of interest would be used to develop the cumulative distribution. It might also be desirable to determine the cumulative distribution for each specific user. It might also be true that specific scenarios continue to show that the Figure of Merit of one adaptive system indicates performance better than a competitor. At the very least this method of evaluation reduces the data that must be studied (i.e., a few cumulative distribution curves instead of many contour plots) and summarizes it in an objective manner.

This Figure of Merit approach described above is not presently in common use but many adaptive antenna specialists agree that it is better than the present practice of preparing countless radiation patterns or contour plots.

#### Summary and Concluding Remarks

The foregoing attempts to describe the necessary ingredients of an adaptive antenna with specific application to a satellite communication system. The basic structure of the weight determining algorithms are discussed covering scenarios from those where the user-interference source locations are known to those where only a characteristic of the user's signal is known. It is pointed out that essentially all adaptive algorithms strive to achieve steady state weights ( $\bar{w}_{opt} = R^{-1} \cdot \bar{w}_0$ ) that are determined by the quiescent, or unstressed, radiation characteristics and the covariance matrix of the array element, or MBA beam port, signals. Finally, a method of evaluating

the performance of an adaptive antenna system is presented.

At least one other popular algorithm which does not fall into the class of those discussed was not included because of this author's unfamiliarity with its formal characteristics. The method consists of applying a random search to a first approximation of the desired weights. The first approximation is usually in accordance with the basic algorithms discussed here.

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